

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question.

- 1- 1-  $\int \text{Sec}x \text{Tan}x \, dx = ?$   
 (A)  $\text{Sec}x + c$  ● (B)  $\text{Sec}^2x + c$  (C)  $\text{Tan}x + c$  (D)  $\ln |\text{Sec}x + \text{tan}x| + c$
- 2- The focus of parabola  $x^2 = -16y$  is  
 (A)  $(0, -4)$  ● (B)  $(0, 0)$  (C)  $(4, 0)$  (D)  $(-4, 0)$
- 3-  $\int_0^2 |x| \, dx$  is  
 (A) 0 (B) 1 (C) 2 ● (D) 4
- 4- Derivative of  $y = f(x)$  at  $x = a$  represents slope of  
 (A) tangent line at  $x = a$  ● (B) secant line (C) perpendicular line (D) straight line
- 5- Projection of vector  $\underline{v}$  along vector  $\underline{u}$  is  
 (A)  $\frac{\underline{u} \cdot \underline{v}}{|\underline{u}|}$  ● (B)  $\frac{\underline{u} \cdot \underline{v}}{|\underline{v}|}$  (C)  $\frac{\underline{u} \cdot \underline{u}}{|\underline{u}|}$  (D)  $\frac{\underline{v} \cdot \underline{v}}{|\underline{v}|}$
- 6- Which one is true?  
 (A)  $i \times i = i$  (B)  $i \cdot i = i$  (C)  $\underline{k} \times \underline{k} \neq 0$  ● (D)  $\underline{k} \times \underline{i} = -j$
- 7- Which one equation represents a circle?  
 (A)  $y^2 = 8x$  (B)  $3x^2 + 3y^2 = 9$  ● (C)  $3x^2 + 5y^2 = 9$  (D)  $x^2 - 2y = 0$
- 8- Which one is point-slope form of a straight line?  
 (A)  $y = mx + c$  (B)  $y - y_1 = m(x - x_1)$  ● (C)  $\frac{x}{a} + \frac{y}{b} = 1$  (D)  $\frac{x}{a} - \frac{y}{b} = 1$
- 9- Order of differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$  is  
 (A) 1 (B) 0 (C) 2 ● (D) 3
- 10- The interval in which  $f(x) = 4 - x^2$ ;  $x \in (-2, 2)$  is increasing  
 (A)  $(0, 2)$  (B)  $(-2, 0)$  ● (C)  $(-2, 2)$  (D)  $(0, 1)$
- 11- The function  $f(x) = \frac{x^2 - 1}{x - 1}$  is not defined at  
 (A)  $x = 0$  (B)  $x = 1$  ● (C)  $x = 2$  (D)  $x = -1$
- 12- If  $f(x) = x^{2/3}$ , the  $f'(8)$  is  
 (A) 3 (B)  $\frac{1}{3}$  ● (C)  $\frac{2}{3}$  (D)  $\frac{1}{2}$
- 13-  $\int \frac{f'(x)}{f(x)} \, dx = ?$   
 (A)  $\ln|x| + c$  (B)  $\ln|f(x)| + c$  ● (C)  $\ln|f'(x)| + c$  (D)  $\ln f(x) \cdot f'(x) + c$
- 14- Slope of the line passing through the points  $(0, -1)$  and  $(7, -15)$  is  
 (A) 2 (B) 0 (C) 1 (D) -2 ●
- 15-  $\lim_{x \rightarrow \infty} (e^x) = ?$   
 (A)  $\infty$  ● (B)  $-\infty$  (C) 1 (D) 0
- 16-  $[\underline{u} \, \underline{v} \, \underline{v}]=?$   
 (A) 1 (B) -1 (C) 0 ● (D)  $\underline{v}$
- 17- Which point is not solution of inequality  $x - 2y \leq 6$   
 (A)  $(1, 4)$  (B)  $(0, -1)$  (C)  $(14, 0)$  ● (D)  $(-4, 0)$
- 18- Major axis of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with  $(a > b)$  is  
 (A)  $x = 0$  (B)  $y = 0$  ● (C)  $x = 1$  (D)  $y = 1$
- 19- Derivative of  $\text{Tan}^{-1}x$  w.r.t.  $x$  is  
 (A)  $\frac{1}{1-x^2}$  (B)  $\frac{1}{x^2-1}$  (C)  $\frac{1}{1+x^2}$  ● (D)  $1+x^2$
- 20- Distance of line  $5x + 12y + 39 = 0$  from origin is  
 (A) 3 ● (B) 5 (C) 12 (D) 39



**Note: Section I is compulsory. Attempt any three (3) questions from Section II.**

**SECTION I**



(2 x 8 = 16)

**2. Write short answers to any EIGHT questions:**

- i- Let  $f(x) = x^2 - x$ , find the value of  $f(x - 1)$ .
- ii- State the domain and range of  $f^{-1}$  if  $f(x) = \frac{1}{x+3}$
- iii- Evaluate  $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$
- iv- Express  $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^{2n}$  in term of e.
- v- Differentiate  $\frac{x^2+1}{x^2-3}$  w.r.t. 'x'
- vi- Find  $\frac{dy}{dx}$  if  $x = at^2$  and  $y = 2at$
- vii- Prove that  $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
- viii- Differentiate  $(\cos\sqrt{x} + \sqrt{\sin x})$  w.r.t 'x'
- ix- Find  $\frac{dy}{dx}$  if  $y = \sin h^{-1}(ax + b)$
- x- Find  $\frac{dy}{dx}$  if  $y = \log_{10}(ax^2 + bx + c)$
- xi- Find  $f'(x)$  if  $f(x) = \frac{e^x}{e^{-x} + 1}$
- xii- Define a stationary point.

**3. Write short answers to any EIGHT questions:**

(2 x 8 = 16)

- i- Use differential to find  $\frac{dy}{dx}$ , if  $xy - \ln x = c$
- ii- Evaluate  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx, (x > 0)$
- iii- Find  $\int \sec x dx$
- iv- Integrate  $\int \sin^{-1} x dx$
- v- Evaluate  $\int e^x (\cos x - \sin x) dx$
- vi- Calculate  $\int_1^2 \frac{x}{x^2+2} dx$
- vii- Solve the differential equation  $\frac{dy}{dx} = \frac{1-x}{y}$
- viii- Find an equation of vertical line through  $(-5, 3)$ .
- ix- Write the equation of line in two intercepts form.
- x- Convert  $15y - 8x + 3 = 0$  in slope intercept form.
- xi- Find the equation of line passing through  $A(-6, 5)$  having slope 7.
- xii- Show that the points  $A(-1, 2)$ ,  $B(7, 5)$  and  $C(2, -6)$  are vertices of right triangle.

(Turn over)

4. Write short answers to any NINE questions:

- i- What is feasible region?
- ii- Derive equation of circle in standard form.
- iii- Write an equation of circle with centre  $(-3, 5)$  and radius 7.
- iv- Check the position of point  $(5, 6)$  with respect to circle:  $2x^2 + 2y^2 + 12x - 8y + 1 = 0$
- v- Find equation of hyperbola with foci  $(0, \pm 9)$ , directrices  $y = \pm 4$ .
- vi- Find the focus and directrix of the parabola if  $x^2 = 5y$ .
- vii- Find an equation of ellipse with foci  $(\pm 3, 0)$  and minor axis length 10.
- viii- Indicate the solution set of system of linear inequality by shading  $4x - 3y \leq 12$  ;  $x \geq -\frac{3}{2}$
- ix- Define equal vector, give an example.
- x- Find the magnitude and direction cosines of  $\underline{v} = 4\hat{i} - 5\hat{j}$
- xi- Find scalar " $\alpha$ " so that the vectors  $2\hat{i} + \alpha\hat{j} + 5\hat{k}$  and  $3\hat{i} + \hat{j} + \alpha\hat{k}$  are perpendicular.
- xii- Which vectors, if any, are parallel or perpendicular  
 $\underline{u} = \hat{i} + 2\hat{j} - \hat{k}$  ,  $\underline{v} = -\hat{i} + \hat{j} + \hat{k}$  ,  $\underline{w} = \frac{-\pi}{2}\hat{i} - \pi\hat{j} + \frac{\pi}{2}\hat{k}$
- xiii- Prove that the vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  ,  $-2\hat{i} + 3\hat{j} - 4\hat{k}$  and  $\hat{i} - 3\hat{j} + 5\hat{k}$  are coplanar.

**SECTION II**

- 5- (a) Evaluate  $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$  5
- (b) If  $\tan y(1 + \tan x) = 1 - \tan x$  , show that  $\frac{dy}{dx} = -1$  5
- 6- (a) If  $x = \sin \theta, y = \sin m\theta$  , show that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$  5
- (b) Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$  5
- 7- (a) Evaluate  $\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx$  5
- (b) Maximize  $f(x, y) = 2x + 5y$  , subject to the constraints  $2y - x \leq 8$  ;  $x - y \leq 4$  ;  $x \geq 0$  ;  $y \geq 0$ . 5
- 8- (a) Find the length of the chord cut off from the line  $2x + 3y = 13$  by the circle  $x^2 + y^2 = 26$ . 5
- (b) Prove that in any  $\Delta ABC$  ,  $b^2 = c^2 + a^2 - 2ca \cos B$  5
- 9- (a) Find the interior angles of a triangle with vertices  $A(-2, 11)$  ,  $B(-6, -3)$  and  $C(4, -9)$  5
- (b) Find the centre, foci, eccentricity, vertices and directrices of the Ellipse  $x^2 + 4y^2 = 16$  5

# Gujranwala Board-2024

Roll No. of Candidate \_\_\_\_\_

**MATHEMATICS**  
**Time: 30 Minutes**

**Intermediate Part-II, Class 12<sup>th</sup> (1<sup>st</sup> A 424- IV)**  
**OBJECTIVE**  
**Code: 8198**

**GROUP: II**  
**PAPER: II**  
**Marks: 20**

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question.



- 1- 1- Differential of  $\sqrt{x}$  is  
 (A)  $\frac{1}{\sqrt{x}} dx$  (B)  $\frac{2}{\sqrt{x}} dx$  (C)  $\frac{1}{2\sqrt{x}} dx$  ● (D)  $\frac{-1}{\sqrt{x}} dx$
- 2- If  $a = b$  then equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  represent  
 (A) Ellipse (B) Circle ● (C) Parabola (D) Hyperbola
- 3- Degree of differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3x = 0$  is  
 (A) 0 (B) 2 (C) 1 ● (D) 3
- 4-  $\frac{d}{dx}(\sin \ln x) = ?$   
 (A)  $\frac{e^x - e^{-x}}{2}$  ● (B)  $\frac{e^x + e^{-x}}{2}$  (C)  $e^x - e^{-x}$  (D)  $e^x + e^{-x}$
- 5- Magnitude of a vector  $\underline{v} = -\underline{i} + \underline{j}$  is  
 (A) a (B)  $\sqrt{2}$  ● (C)  $\sqrt{2}$  (D)  $\sqrt{3}$
- 6- If dot product of two non-zero vectors is zero then vectors will be  
 (A) perpendicular ● (B) parallel (C) collinear (D) all of these
- 7- Length of latus ractum of parabola  $y^2 = 4ax$  is  
 (A) 2a (B) 4ax (C) 4a ● (D)  $\frac{1}{2a}$
- 8- Every homogeneous equation  $ax^2 + 2hxy + by^2 = 0$  represent two real lines through origin if  
 (A)  $h^2 - ab < 0$  (B)  $h^2 - ab > 0$  (C)  $h^2 = ab$  ● (D) both (B) and (C)
- 9- If  $\alpha$  is constant then  $\int \cot \alpha \cdot dy$  is  
 (A)  $\sin \alpha + c$  (B)  $-\sin \alpha + c$  (C)  $x \sin \alpha + c$  (D)  $y \cot \alpha + c$  ●
- 10- If  $f(x) = \cos x$ , then  $f'\left(\frac{\pi}{2}\right)$  is  
 (A) -1 ● (B) 1 (C) 0 (D)  $\frac{1}{2}$
- 11-  $\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = ?$   
 (A)  $3a^2$  ● (B)  $a^2$  (C) 0 (D) un-defined
- 12- Derivative of  $\sqrt{x}$  at  $x = a$  is  
 (A)  $\frac{1}{\sqrt{a}}$  (B)  $-\frac{1}{2\sqrt{a}}$  (C)  $\frac{1}{2\sqrt{a}}$  ● (D)  $2\sqrt{a}$

(Turn over)

## Gujranwala Board-2024

(2)

- 13-  $\int \frac{\ln x}{x} dx$  is equal to  
(A)  $\ln(\ln x) + c$  (B)  $\frac{(\ln x)^2}{2} + c$  ● (C)  $\ln x + c$  (D)  $\frac{\ln x}{2} + c$
- 14- Slope intercept form of a line is  
(A)  $y = mx + c$  ● (B)  $\frac{x}{a} + \frac{y}{b} = 1$  (C)  $x = 0$  (D)  $y = 0$
- 15- The function  $f(x) = \frac{2+3x}{2x}$  is not continuous at  
(A)  $x = 3$  (B)  $x = 0$  ● (C)  $x = -\frac{2}{3}$  (D)  $x = 1$
- 16-  $\frac{1}{6}[u \ v \ w]$  is formula to calculate  
(A) area of triangle (B) volume of parallelepiped  
(C) volume of tetrahedron ● (D) area of parallelogram
- 17- (2, 1) is solution of in-equality  
(A)  $2x + y > 5$  (B)  $x - 2y > 1$  (C)  $3x - 5y < 7$  ● (D)  $2x + y < 5$
- 18- Eccentricity of hyperbola is  
(A)  $e < 1$  (B)  $e = 0$  (C)  $e = 1$  (D)  $e > 1$  ●
- 19-  $\frac{d}{dx} \left[ \frac{1}{g(x)} \right]$  is equal to  
(A)  $\frac{1}{[g(x)]^2}$  (B)  $\frac{-g'(x)}{g(x)}$  (C)  $\frac{-1}{[g(x)]^2}$  (D)  $\frac{-g'(x)}{[g(x)]^2}$  ●
- 20- Distance of point  $(\cos 3x, \sin 3x)$  from origin is  
(A) 9 (B) 6 (C) 3 (D) 1 ●



314-(IV)-1<sup>st</sup>A 424-24000

**SUBJECTIVE**

**Note: Section I is compulsory. Attempt any three (3) questions from Section II.**

SECTION I



**2. Write short answers to any EIGHT questions:**

**(2 x 8 = 16)**

- i- Define rational function. Give one example also.
- ii- Find  $\text{gof}(x)$ , when  $f(x) = \sqrt{x+1}$  ;  $g(x) = \frac{1}{x^2}$  ,  $x \neq 0$
- iii- Evaluate  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$
- iv- Find 'c' so that  $\lim_{x \rightarrow -1} f(x)$  exists, when  $f(x) = \begin{cases} x+2 & , x \leq -1 \\ c+2 & , x > -1 \end{cases}$
- v- Differentiate  $(x^2 + 5)(x^3 + 7)$  w.r.t x.
- vi- Find derivative of  $\tan^3 \theta \sec^2 \theta$  w.r.t  $\theta$ .
- vii- Find  $\frac{dy}{dx}$ , if  $y = \sinh^{-1}\left(\frac{x}{2}\right)$
- viii- Define critical value and critical point of function f.
- ix- Differentiate  $\cot^{-1}\left(\frac{x}{a}\right)$  w.r.t x.
- x- Find derivative of  $\frac{x^2+1}{x^2-3}$  w.r.t x.
- xi- State product rule for derivative of two functions.
- xii- Differentiate  $\sin^2 x$  w.r.t  $\cos^4 x$ .

**3. Write short answers to any EIGHT questions:**

**(2 x 8 = 16)**

- i- Find  $\delta y$  if  $y = x^2 - 1$  and x changes from 3 to 3.02
- ii- Evaluate  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- iii- Evaluate  $\int \frac{dx}{x(\ln 2x)^3}$  ( $x > 0$ )
- iv- Evaluate  $\int x \tan^2 x dx$
- v- Evaluate  $\int \frac{e^x(1+x)}{(2+x)^2} dx$
- vi- Evaluate  $\int_0^{\pi/6} x \cos x dx$
- vii- Solve the differential equation  $\sin y \operatorname{Cosec} x \frac{dy}{dx} = 1$
- viii- Find the distance and midpoint of line joining A(-8 , 3) and B(2 , -1).
- ix- Find an equation of line with x-intercept:-9 and slope:-4
- x- Transform the equation  $5x - 12y + 39 = 0$  into slope intercept form.
- xi- Determine the value of P such that the lines  $2x - 3y - 1 = 0$  ,  $3x - y - 5 = 0$  and  $3x + Py + 8 = 0$  meet at a point.
- xii- Find the angle between the lines represented by  $x^2 - xy - 6y^2 = 0$

**(Turn over)**

# Gujranwala Board-2024

(2)



(2 x 9 = 18)

4. Write short answers to any NINE questions:

- i- Define feasible region.
- ii- Graph the feasible region of inequality  $3x + 2y \geq 6$  ,  $x \geq 0$  ,  $y \geq 0$
- iii- Write an equation of circle with centre (5, -2) and radius 4.
- iv- Write down equation of tangent to  $x^2 + y^2 = 25$  at (4, 3)
- v- Find the focus and vertex of parabola  $y^2 = 8x$
- vi- Write equation of the ellipse whose foci  $(\pm 3, 0)$  and minor axis of length 10.
- vii- Find the foci and eccentricity of  $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- viii- Find the length of tangent drawn from point (-5, 4) to the circle  $x^2 + y^2 - 2x + 3y - 26 = 0$
- ix- Find a unit vector in the same direction of the vector  $\underline{v} = [3, -4]$
- x- Write the direction cosine of vector  $\underline{v} = -\hat{i} + \hat{j} + \hat{k}$
- xi- Find a scalar ' $\alpha$ ' so that vectors  $2\hat{i} + \alpha\hat{j} + 5\hat{k}$  and  $3\hat{i} + \hat{j} + \alpha\hat{k}$  are perpendicular.
- xii- If  $\underline{a} = 4\hat{i} + 3\hat{j} + \hat{k}$  and  $\underline{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ , find  $|\underline{a} \times \underline{b}|$
- xiii- A force  $\underline{F} = 4\hat{i} - 3\hat{k}$  passes through A(2, -2, 5). Find its moment about B(1, -3, 1).

## SECTION II

- 5- (a) Evaluate :  $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$  5
- (b) Differentiate :  $\sec^{-1}\left(\frac{x^2 + 1}{x^2 - 1}\right)$  w.r.t "x" 5
- 6- (a) If  $y = e^x \sin x$ ; show that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$  5
- (b) Evaluate :  $\int \operatorname{Cosec}^3 x \, dx$  5
- 7- (a) Evaluate :  $\int_0^{\pi/4} \frac{\sin x - 1}{\cos^2 x} \, dx$  5
- (b) Graph the feasible region of the following system of linear inequalities and find the corner points  $2x - 3y \leq 6$   
 $2x + 3y \leq 12$   
 $x \geq 0, y \geq 0$  5
- 8- (a) Find an equation of the circle passing through the points A(1, 2) and B(1, -2) and touching the line  $x + 2y + 5 = 0$  5
- (b) Use vectors, to prove that the diagonals of a parallelogram bisect each other. 5
- 9- (a) Find the equation of perpendicular bisector of a segment joining the points A(3, 5) and B(9, 8). 5
- (b) Find the equation of parabola with focus (-3, 1) and directrix  $x = 3$ . 5

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# Gujranwala Board-2023


Roll No. of Candidate \_\_\_\_\_

**MATHEMATICS**  
Time: 30 Minutes

**Intermediate Part II Class 12<sup>th</sup> (1<sup>st</sup>A 423-III)**  
**OBJECTIVE**  
Code: 8195

**GROUP: I**  
**PAPER: II**  
**Marks: 20**

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question.

- 1- 1- Equation of horizontal line through (3, 1)  
(A)  $x = 3$       (B)  $x = 1$       (C)  $y = 3$       (D)  $y = 1$  
- 2-  $\int \frac{1}{x} dx =$  \_\_\_\_\_  
(A)  $\ln x + c$       (B)  $-\frac{1}{x^2} + c$       (C)  $-\frac{1}{x} + c$       (D)  $e^x + c$
- 3- It is not unit vector  
(A)  $[1, 0, 0]$       (B)  $[0, 1, 0]$       (C)  $[1, 1, 1]$       (D)  $[0, 0, 1]$
- 4- Eccentricity  $e$  of hyperbola is  
(A)  $e < 1$       (B)  $e > 1$       (C)  $e = 1$       (D)  $e = 0$
- 5- Focus of parabola  $x^2 = -16y$  is \_\_\_\_\_  
(A)  $(4, 0)$       (B)  $(-4, 0)$       (C)  $(0, 4)$       (D)  $(0, -4)$
- 6-  $\frac{d}{dx} \sqrt{x} =$  \_\_\_\_\_  
(A)  $\frac{1}{2}$       (B)  $\frac{1}{2\sqrt{x}}$       (C)  $\frac{1}{2\sqrt{x}}$       (D)  $\frac{2}{\sqrt{x}}$
- 7-  $\frac{d}{dx} \left( \frac{1}{x^2} \right)$  at  $x=1$  is \_\_\_\_\_  
(A) 2      (B) -2      (C) 1      (D) -1
- 8-  $f(x) = 2x^2 + 4x - 2$ , then  $f(-2) =$  \_\_\_\_\_  
(A) 0      (B) -1      (C) 2      (D) -2
- 9-  $\int_0^1 \frac{1}{1+x^2} dx =$  \_\_\_\_\_  
(A)  $\pi$       (B)  $\frac{\pi}{2}$       (C)  $\frac{\pi}{3}$       (D)  $\frac{\pi}{4}$
- 10- Distance of point  $(-2, -3)$  from x-axis is  
(A) 2      (B) -2      (C) 3      (D) -3
- 11- Radius of circle  $x^2 + y^2 - 4x + 6y + 9 = 0$  is  
(A) 2      (B) 3      (C) 4      (D) 9
- 12- If  $f(x)$  has maximum value at  $x = c$ , then  $f'(c) = 0$  but  $f''(x)$  is \_\_\_\_\_  
(A) negative      (B) positive      (C) zero      (D) undefined

(Turn over)



## Gujranwala Board-2023

(2)

- 13- Which one is constant function  
(A)  $f(x) = x$  (B)  $f(x) = x^2$  (C)  $f(x) = 5$  (D)  $f(x) = \sin x$
- 14- Vectors  $\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k}$  and  $\vec{b} = \vec{i} - \vec{j} - x\vec{k}$  are perpendicular, then value of  $x$  is \_\_\_\_\_  
(A)  $-5$  (B)  $5$  (C)  $1$  (D)  $-1$
- 15- Length of major axis of  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  is  
(A)  $10$  (B)  $5$  (C)  $8$  (D)  $4$
- 16-  $x = 2$  is solution of the inequality  
(A)  $2x - 1 \leq 0$  (B)  $2x - 1 \geq 0$  (C)  $x - 1 \leq 0$  (D)  $x + 1 \leq 0$
- 17- The lines represented by  $ax^2 + 2hxy + by^2 = 0$  are orthogonal if  
(A)  $a + b = 1$  (B)  $a - b = 0$  (C)  $a + b = 0$  (D)  $a - b = 1$
- 18- Solution of  $\frac{dy}{dx} = 2x$  is \_\_\_\_\_  
(A)  $y = x^2 + c$  (B)  $y = x + c$  (C)  $y = \ln x + c$  (D)  $y = e^x + c$
- 19-  $\int 2 \sec^2 2x \, dx =$  \_\_\_\_\_  
(A)  $\frac{\tan 2x}{2} + c$  (B)  $\tan 2x + c$  (C)  $\sec 2x + c$  (D)  $\frac{\sec 2x}{2} + c$
- 20-  $\frac{d}{dx} \sinh 2x =$  \_\_\_\_\_  
(A)  $2 \cosh 2x$  (B)  $2 \sinh 2x$  (C)  $-2 \cosh 2x$  (D)  $-2 \sinh 2x$

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

### SECTION I

2. Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Show that the parametric equations  $x = a\cos\theta$ ,  $y = b\sin\theta$  represent the equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ii- Let the real valued functions 'f' and 'g' be defined by  $f(x) = 2x+1$  and  $g(x) = x^2-1$ , obtain the expressions  $f \circ g(x)$  and  $f^2(x)$

iii- Evaluate the limit  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

iv- Differentiate w.r.t.x  $\frac{2x-1}{\sqrt{x^2+1}}$

v- Find  $\frac{dy}{dx}$  if  $x = at^2$  and  $y = 2$  at

vi- Find  $\frac{dy}{dx}$  if  $4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

vii- If  $\tan y(1 + \tan x) = 1 - \tan x$ , show that  $\frac{dy}{dx} = -1$

viii- Find  $y_2$  if  $y = \ln\left(\frac{2x+3}{3x+2}\right)$

ix- Determine the intervals in which f is increasing or decreasing for the domain mentioned.  
 $f(x) = \sin x$ ;  $x \in (-\pi, \pi)$

x- Find two positive integers whose sum is 30 and their product will be maximum.

xi- Define feasible region and feasible solution.

xii- Graph the feasible region of the following system of linear inequalities and find the corner points

$$x + y \leq 5$$

$$-2x + y \geq 2$$

$$x \geq 0$$

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

i- Find  $\delta y$  if  $y = x^2 - 1$  and  $x$  changes from 3 to 3.02

ii- Evaluate  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$

iii- Find the anti-derivative of  $x^2 \ln x$

iv- Evaluate  $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

v- Evaluate  $\int_{\pi/6}^{\pi/3} \cos t dt$

vi- Find the area between x-axis and the curve  $y = \sin 2x$  from  $x = 0$  to  $x = \frac{\pi}{3}$

vii- Solve the differential equation  $\frac{dy}{dx} = \frac{1+y^2}{e^{-x}}$

viii- If  $\underline{v} = 3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$  and  $\underline{w} = 5\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  then find  $|3\underline{v} + \underline{w}|$

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(2)

- ix- Find direction cosines of vector  $\vec{PQ}$  where  $P(2,1,5)$  and  $Q(1,3,1)$
- x- Find a vector perpendicular to each of the vectors  $\underline{u} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\underline{v} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
- xi- Prove that  $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$
- xii- Calculate the projection of  $\underline{a} = \mathbf{i} - \mathbf{k}$  along  $\underline{b} = \mathbf{j} + \mathbf{k}$

**4. Write short answers to any NINE questions:**

(2 x 9 = 18)

- i- Find the point three-fifth of the way along the line segment from  $A(-5, 8)$  to  $B(5, 3)$
- ii- By means of slopes show that the points  $(-4, 6)$ ,  $(3, 8)$  and  $(10, 10)$  lie on the same line.
- iii- Find an equation of line with x-Intercept =  $-9$  and slope is  $-4$
- iv- Find measure of angle between the lines represented by  $10x^2 - 23xy - 5y^2 = 0$
- v- Find  $h$  such that the points  $A(-1, h)$ ,  $B(3, 2)$  and  $C(7, 3)$  are collinear.
- vi- Find an equation of the line through  $(11, -5)$  and parallel to a line with slope  $-24$ .
- vii- Find the co-ordinates of the point that divides the join of  $A(-6, 3)$  and  $B(5, -2)$  externally in ratio  $2:3$
- viii- Find centre and radius of the circle  $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
- ix- Write down an equation of the parabola with focus  $(2, 5)$  and directrix  $y = 1$
- x- Find an equation of circle of radius  $a$  and lying in 2nd Quadrant such that it is tangent to both the axes.
- xi- Find focus, vertex of the parabola  $x^2 = 4(y-1)$
- xii- Find an equation of the hyperbola with given foci  $(0, \pm 6)$ ,  $e = 2$
- xiii- Find centre and foci of the hyperbola  $\frac{y^2}{4} - x^2 = 1$

**SECTION II**

Note: Attempt any three (3) questions.

- 5- (a) Express the limit in terms of  $e$   $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ ,  $x > 0$  5
- (b) Find  $\frac{dy}{dx}$  of the parametric equations  $x = \frac{a(1-t^2)}{1+t^2}$ ,  $y = \frac{2bt}{1+t^2}$  5
- 6- (a) Show that  $\int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2} \sqrt{a^2 - x^2} + c$  5
- (b) Find an equation of the line through the point  $(2, -9)$  and intersection of the lines  $2x + 5y - 8 = 0$  and  $3x - 4y - 6 = 0$  5
- 7- (a) Evaluate  $\int_0^{\pi/4} \cos^4 t dt$  5
- (b) Maximize  $f(x, y) = 2x + 5y$  subject to the constraints  $2y - x \leq 8$ ;  $x - y \leq 4$ ;  $x \geq 0$ ;  $y \geq 0$  5
- 8- (a) If  $y = (\cos^{-1}x)^2$ , prove that  $(1 - x^2)y_2 - xy_1 - 2 = 0$  5
- (b) Write down an equation of the circle that passes through the given points  $A(-7, 7)$ ,  $B(5, -1)$ ,  $C(10, 0)$  5
- 9- (a) Find centre, foci, eccentricity, vertices and directrices of  $x^2 + 16x + 4y^2 - 16y + 76 = 0$  5
- (b) Prove that in any  $\triangle ABC$ ;  $a = b \cos C + c \cos B$  5

# Gujranwala Board-2023

Roll No. of Candidate \_\_\_\_\_

**MATHEMATICS**

**Intermediate Part-II, Class 12<sup>th</sup> (1<sup>st</sup>A 423- II)**

**GROUP: II**

**Time: 30 Minutes**

**OBJECTIVE**

**PAPER: II**

**Code: 8194**

**Marks: 20**

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question.



- 1- 1-  $\hat{i} \cdot (2\hat{j} \times \hat{k}) =$   
(A) 0 (B) 2 (C) 4 (D) 6
- 2- The co-ordinates of vertex of parabola  $x+8-y^2+2y=0$  will be  
(A) (-9,1) (B) (9,1) (C) (9,-1) (D) (-9,-1)
- 3- Mid-point of hypotenuse of a right triangle is called as  
(A) circumcentre (B) incentre (C) orthocentre (D) centroid
- 4-  $x=0$  is the solution of inequality  
(A)  $3x-2 > 0$  (B)  $3x+5 < 0$  (C)  $2x-6 < 0$  (D)  $x+3 < 0$
- 5- If a line intersects y-axis at (0, a), then 'a' is called  
(A) x-intercept (B) y-intercept (C) inclination (D) slope
- 6-  $\int \sin 2x \, dx =$   
(A)  $-\frac{\cos 2x}{2}$  (B)  $\frac{\cos 2x}{2}$  (C)  $2 \cos 2x$  (D)  $-2 \cos 2x$
- 7-  $\int \tan x \, dx =$   
(A)  $\ln \cos x$  (B)  $\ln |\sec x|$  (C)  $\ln \sin x$  (D)  $\ln |\cot x|$
- 8- If  $f(x) = \sin x$ , then  $f'(\pi) =$   
(A) -1 (B) 1 (C) 0 (D)  $\frac{1}{2}$
- 9-  $\frac{d}{dx} \left( \frac{2}{x} \right) =$   
(A)  $\ln |x^2|$  (B)  $\frac{-2}{x^2}$  (C)  $-2x^2$  (D)  $2^x$
- 10-  $\lim_{x \rightarrow 3} (2x+4) =$   
(A) 3 (B) 6 (C) 10 (D) 12
- 11-  $\cos \theta =$   
(A)  $\hat{a} \cdot \hat{b}$  (B)  $|\underline{a} \times \underline{b}|$  (C)  $\underline{a} \times \underline{b}$  (D)  $\sin \theta$
- 12- The focus of parabola  $y^2 = 4ax$  is  
(A) (0, a) (B) (-a, 0) (C) (a, 0) (D) (0, -a)

**(Turn over)**

## Gujranwala Board-2023

(2)

- 13- Two circles are said to be concentric if they have same \_\_\_\_\_.  
(A) radius (B) diameter (C) center (D) length
- 14- If a line is parallel to x-axis, then inclination =  
(A)  $0^\circ$  (B)  $30^\circ$  (C)  $45^\circ$  (D)  $90^\circ$
- 15-  $\int \sqrt{x} dx =$   
(A)  $\frac{\sqrt{x}}{2}$  (B)  $\frac{x\sqrt{x}}{3}$  (C)  $\frac{1}{2\sqrt{x}}$  (D)  $\frac{2x\sqrt{x}}{3}$
- 16-  $y = mx + c$  is \_\_\_\_\_ form of equation of line.  
(A) normal (B) point-slope (C) slope-intercept (D) intercept
- 17- If  $f(x) = \sqrt{x-12}$ , then  $f(16) =$   
(A) 16 (B) 12 (C) 28 (D) 2
- 18- If  $y = \ln(\sin x)$ , then  $\frac{dy}{dx} =$   
(A)  $\tan x$  (B)  $\cot x$  (C)  $-\tan x$  (D)  $-\cot x$
- 19- If  $y = \cosh 2x$ , then  $\frac{dy}{dx} =$   
(A)  $2\sinh 2x$  (B)  $-\sinh 2x$  (C)  $-2\sinh 2x$  (D)  $\cosh 2x$
- 20-  $\int_0^{\pi/2} \cos x dx =$   
(A) 2 (B) 0 (C) -1 (D) 1

**SUBJECTIVE**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

**SECTION I**



(2 x 8 = 16)

**2. Write short answers to any EIGHT questions:**

- i- Prove that  $\operatorname{sech}^2 x = 1 - \operatorname{Tanh}^2 x$
- ii- Evaluate  $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x} - \sqrt{3}}$
- iii- Find  $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$
- iv- If  $y = x^4 + 2x^2 + 2$ , prove that  $\frac{dy}{dx} = 4x\sqrt{y-1}$
- v- Differentiate  $\sin x$  w.r.t  $\cot x$
- vi- If  $y = \cot^{-1}\left(\frac{x}{a}\right)$ , find  $\frac{dy}{dx}$
- vii- If  $f(x) = \ln(e^x + e^{-x})$ , find  $f'(x)$
- viii- If  $y = \operatorname{Tanh}^{-1}(\sin x)$ , find  $\frac{dy}{dx}$
- ix- If  $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ , find  $y_2$
- x- Find the interval in which  $f(x)$  is increasing,  $f(x) = 4 - x^2$ ,  $x \in (-2, 2)$
- xi- Define problem constraints.
- xii- Graph the solution set of linear inequality in  $xy$ -plane,  $3x + 7y \geq 21$

**3. Write short answers to any EIGHT questions:**

(2 x 8 = 16)

- i- Using differentials find  $\frac{dy}{dx}$  if  $xy + x = 4$
- ii- Evaluate  $\int (2x+3)^{1/2} dx$
- iii- Evaluate  $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- iv- Evaluate  $\int e^{-x}(\cos x - \sin x) dx$
- v- Evaluate  $\int_{-1}^2 (x + |x|) dx$
- vi- Find the area bounded by  $\cos$  function from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$
- vii- Solve  $\frac{dy}{dx} = \frac{y}{x^2}$
- viii- If  $O$  is the origin and  $\overline{OP} = \overline{AB}$ , find the point  $P$  when  $A$  and  $B$  are  $(-3, 7)$  and  $(1, 0)$  respectively.
- ix- Find a unit vector in the direction of  $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$
- x- Find  $\alpha$  so that  $\underline{u}$  and  $\underline{v}$  are perpendicular  $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$  and  $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$
- xi- Find a unit vector perpendicular to the plane containing  $\underline{a}$  and  $\underline{b}$ , where  $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$ ,  $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$
- xii- Given a force  $\vec{F} = 2\underline{i} + \underline{j} - 3\underline{k}$  acting at a point  $A(1, -2, 1)$ . Find the moment of  $\vec{F}$  about the point  $B(2, 0, -2)$

(Turn over)

# Gujranwala Board-2023

(2)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- i- Show that the points A(0,2), B( $\sqrt{3}$ ,-1) and C(0,-2) are vertices of a right triangle.
- ii- The two points P(3,2) and O'(1,3) are given in XY-coordinate system. Find the XY-coordinates of P referred to the translated axes O'X and O'Y
- iii- Find K so that the line joining A(7,3), B(K,-6) and the line joining C(-4,5), D(-6,4) are parallel.
- iv- Find an equation of the vertical line through (-5,3)
- v- Find the distance from the point P(6,-1) to the line  $6x - 4y + 9 = 0$
- vi- Find point of intersection of the lines  $x - 2y + 1 = 0$  and  $2x - y + 2 = 0$
- vii- Find measure of the angle between the lines represented by  $x^2 - xy - 6y^2 = 0$
- viii- Find an equation of the circle with centre at (5,-2) and radius 4
- ix- Check the position of the point (5,6) with respect to the circle  $x^2 + y^2 = 81$
- x- Find the focus and vertex of parabola  $x^2 = -16y$
- xi- Find equation of ellipse with foci ( $\pm 3, 0$ ) and minor axis of length 10
- xii- Find the centre and foci of  $x^2 - y^2 = 9$
- xiii- Find the point of intersection of the given conics  $x^2 + y^2 = 8$  and  $x^2 - y^2 = 1$



## SECTION II

Note: Attempt any three (3) questions.

- 5- (a) If  $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & x \neq 2 \\ k & x = 2 \end{cases}$ , Find value of k so that f is continuous at  $x = 2$  5
- (b) Differentiate  $\cos x$  from the first principle. 5
- 6- (a) Evaluate  $\int e^{2x} \cos 3x \, dx$  5
- (b) Find the area of the region bounded by the triangle with vertices (a, b+c), (a, b-c) and (-a, c) 5
- 7- (a) Solve the differential equation  $y - x \frac{dy}{dx} = 2 \left( y^2 + \frac{dy}{dx} \right)$  5
- (b) Minimize  $z = 2x + y$  subject to constraints  $x + y \geq 3$ ,  $7x + 5y \leq 35$ ,  $x \geq 0$ ,  $y \geq 0$  5
- 8- (a) If  $x = a(\theta + \sin \theta)$ ,  $y = a(1 + \cos \theta)$  then, show that  $y^2 = \frac{d^2y}{dx^2} + a = 0$  5
- (b) Find an equation of the circle which passes through the points A(5,10), B(6,9) and C(-2,3) 5
- 9- (a) Find an equation of the ellipse with centre (0,0), major axis horizontal, the points (3,1), (4,0) lie on the graph. 5
- (b) Find the volume of the tetrahedron whose vertices are A(2,1,8), B(3,2,9), C(2,1,4) and D(3,3,10) 5

313-1<sup>st</sup>A 423-22000

# Gujranwala Board-2021

Roll No. of Candidate \_\_\_\_\_

Mathematics  
Time: 30 Minutes

(INTERMEDIATE PART II)-421-(II)

**OBJECTIVE**

**Code: 8193**

**GROUP: I**  
**PAPER: II**  
**Marks: 20**

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker/pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.



- 1- 1- Length of vector  $2\hat{i} - \hat{j} - 2\hat{k}$  is  
(A) 2 (B) 4 (C) 3 (D) 5
- 2- The unit vector along y-axis is  
(A)  $\hat{i}$  (B)  $\hat{j}$  (C)  $\hat{k}$  (D) 1
- 3- Focus of parabola  $x^2 = -16y$  is  
(A) (0,4) (B) (4, 0) (C) (0, -4) (D) (-4, 0)
- 4- If  $a > b$ , then  
(A)  $-a < b$  (B)  $-a < -b$  (C)  $a < -b$  (D)  $a > b$
- 5- The point of intersection of lines  $x + y = 2$  and  $2x - y = 1$  is  
(A) (1, 2) (B) (-1, 2) (C) (-1, -2) (D) (1, 1)
- 6- Order of differential equation  $y \frac{dy}{dx} + 2x = 0$  is  
(A) 2 (B) 3 (C) 4 (D) 1
- 7-  $\int \tan \frac{\pi}{4} dx$  is  
(A)  $\ln \sin \frac{\pi}{4}$  (B)  $x$  (C)  $\sec^2 \frac{\pi}{4}$  (D)  $\frac{x}{4}$
- 8-  $\frac{d^2}{dx^2}(2^x)$  is  
(A)  $x 2^{x-1}$  (B)  $\ln 2^x$  (C)  $2^x (\ln 2)^2$  (D)  $x \ln 2$
- 9-  $\frac{d}{dx}(\sec^{-1} x + \operatorname{cosec}^{-1} x)$  equals  
(A) 1 (B) 2 (C) 3 (D) zero
- 10-  $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$  equals  
(A) zero (B) 1 (C) 2 (D) 3

(Turn over)



# Gujranwala Board-2021

(2)

- 11- Derivative of  $\cot x$  w.r.t.  $x$  is  
(A)  $-\operatorname{cosec}^2 x$  (B)  $\sec^2 x$  (C)  $+\operatorname{cosec}^2 x$  (D)  $-\sec^2 x$
- 12-  $(\underline{i} \times \underline{j}) \times \underline{k}$  equals  
(A)  $-1$  (B)  $1$  (C) zero (D)  $2$
- 13-  $\int_0^1 \frac{1}{1+x^2} dx$  equals  
(A)  $\frac{\pi}{4}$  (B)  $\frac{2\pi}{3}$  (C)  $\frac{3\pi}{4}$  (D)  $\pi$
- 14- Directrices of  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  are  
(A)  $x = \pm \frac{c}{e^2}$  (B)  $y = \pm \frac{c}{e^2}$  (C)  $x = \pm \frac{c}{e}$  (D)  $\pm \frac{e^2}{c}$
- 15- The lines  $\ell_1, \ell_2$  with slopes  $m_1, m_2$  are perpendicular if  
(A)  $m_1 m_2 = 1$  (B)  $m_1 = m_2$  (C)  $m_1 m_2 = -1$  (D)  $m_1 + m_2 = 0$
- 16- Differential of  $y$  is  
(A)  $dy'$  (B)  $\frac{dy}{dx}$  (C)  $dy$  (D)  $dx$
- 17-  $\frac{d}{dx} (\cos \sqrt{x})$   
(A)  $\frac{-\sin \sqrt{x}}{\sqrt{x}}$  (B)  $-\sin \sqrt{x}$  (C)  $\frac{-\sin \sqrt{x}}{2\sqrt{x}}$  (D)  $\frac{\cos \sqrt{x}}{\sqrt{x}}$
- 18- Function  $F(x) = \frac{3x}{x^2+1}$  is called  
(A) even function (B) odd function (C) constant function (D) linear function
- 19- Slope of line parallel to  $x$ -axis is  
(A)  $-1$  (B) zero (C)  $1$  (D)  $2$
- 20- Length of diameter of circle  $x^2 + y^2 = 9$  is  
(A)  $6$  (B)  $3$  (C)  $9$  (D)  $4$

312-(II)-421-28000

**SUBJECTIVE**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

**SECTION I**



2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Show that the parametric equations  $x = at^2$ ,  $y = 2at$  represent the equation of parabola  $y^2 = 4ax$
- ii- Find  $g \circ f(x)$  if  $f(x) = \frac{1}{\sqrt{x-1}}$ ,  $x \neq 1$ ,  $g(x) = (x^2 + 1)^2$
- iii- Evaluate  $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- iv- Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$
- v- Find the derivative of  $\sqrt{x}$  at  $x = a$  from first principle.
- vi- Differentiate  $\frac{2x-3}{2x+1}$  w. r. t. 'x'.
- vii- Find  $\frac{dy}{dx}$  if  $y^2 + x^2 - 4x = 5$
- viii- Differentiate  $\cos \sqrt{x} + \sqrt{\sin x}$  w. r. t. 'x'.
- ix- Find  $\frac{dy}{dx}$  if  $y = \log_{10}(ax^2 + bx + c)$
- x- Find  $\frac{dy}{dx}$  if  $y = \frac{x}{\ln x}$
- xi- Find  $y_2$  if  $y = x^2 \cdot e^{-x}$
- xii- Determine the interval in which  $f(x) = \cos x$ ;  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , is increasing.

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Using differentials find  $\frac{dy}{dx}$  and  $\frac{dx}{dy}$  for  $xy + x = 4$
- ii- Integrate  $\frac{1}{\sqrt{x+a} + \sqrt{x}}$  w.r.t x
- iii- Solve  $\int \operatorname{cosec} x \, dx$
- iv- Evaluate  $\int \frac{ax}{\sqrt{a^2 - x^4}} \, dx$
- v- Solve  $\int e^{ax} \left[ a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}} \right] dx$
- vi- Evaluate  $\int_1^2 \frac{x}{x^2+2} \, dx$
- vii- Solve  $\frac{x^2+1}{y+1} = \frac{x}{y} \frac{dy}{dx}$
- viii- Evaluate  $\int_0^{\pi/6} x \cos x \, dx$
- ix- Show that the points  $A(0,2)$ ,  $B(\sqrt{3},-1)$  and  $C(0,2)$  are vertices of a right triangle.
- x- Find the equation of line with slope:  $-5$  and  $y$ -intercept is  $-7$
- xi- Show that the points  $(-1, -3)$ ,  $(1, 5)$  and  $(2,9)$  lie on the same straight line.





**4. Write short answers to any NINE questions:**

- i- Graph the solution set of linear inequality in xy-plane,  $3x - 2y \geq 6$
- ii- Find the equation of circle with centre  $(\sqrt{2}, -3\sqrt{3})$  and radius  $2\sqrt{2}$
- iii- Find the focus and vertex of the parabola  $x^2 - 4x - 8y + 4 = 0$
- iv- Write an equation of parabola with axis  $y = 0$ , through  $(2,1)$  and  $(11,2)$
- v- Find the coordinate of vertices of a hyperbola  $\frac{y^2}{16} - \frac{x^2}{49} = 1$
- vi- Find the foci of the hyperbola  $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- vii- Find the sum of vectors  $\overline{AB}$  and  $\overline{CD}$  given four points  $A(1, -1)$ ,  $B(2,0)$ ,  $C(-1,3)$  and  $D(-2,2)$
- viii- Find a unit vector in the direction of  $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
- ix- Let  $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$ ,  $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$  find  $\underline{v} - 3\underline{w}$
- x- Find a vector whose magnitude is 4 and is parallel to  $2\underline{i} - 3\underline{j} + 6\underline{k}$
- xi- Find the direction cosines of  $\overline{PQ}$  where  $P = (2, 1, 5)$ ,  $Q = (1, 3, 1)$
- xii- If  $\underline{v}$  is a vector for which  $\underline{v} \cdot \underline{i} = 0$ ,  $\underline{v} \cdot \underline{j} = 0$ ,  $\underline{v} \cdot \underline{k} = 0$ , find  $\underline{v}$
- xiii- Find the area of a parallelogram whose vertices are  $A(1, 2, -1)$ ,  $B(4, 2, -3)$ ,  $C(6, -5, 2)$ ,  $D(9, -5, 0)$

SECTION II

- 5- (a) Evaluate limit by using algebraic techniques:  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x^m - a^m}$  5
- (b) Find  $\frac{dy}{dx}$  of the given parametric functions:  $x = \frac{a(1-t^2)}{1+t^2}$ ;  $y = \frac{2bt}{1+t^2}$  5
- 6- (a) Show that  $\int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + c$  5
- (b) Find the area of the region bounded by the triangle whose sides are  $7x - y - 10 = 0$ ,  $10x + y - 41 = 0$ ,  $3x + 2y + 3 = 0$  5
- 7- (a) Solve the given differential equation:  $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1+y^2)$  5
- (b) Maximize  $f(x, y) = 2x + 5y$  subject to the constraints  $2y - x \leq 8$ ,  $x - y \leq 4$ ;  $x \geq 0$ ,  $y \geq 0$  5
- 8- (a) Find the centre and radius of the circle  $4x^2 + 4y^2 - 8x + 12y - 25 = 0$  5
- (b) If  $\underline{a} + \underline{b} + \underline{c} = 0$ , then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$  5
- 9- (a) Show that  $y = \frac{\ell \ln x}{x}$  has maximum value at  $x = e$  5
- (b) Find the centre, foci and vertices of equation  $9x^2 - y^2 - 36x - 6y + 18 = 0$  5

# Gujranwala Board-2019

Roll No. of Candidate \_\_\_\_\_

**Mathematics**

**(INTER PART II)-419-(III)**

**PAPER: II**

**GROUP: I**

**Time: 30 Minutes**

**Code: 8195**

**Marks: 20**

## OBJECTIVE

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.



1- 1- If A(-3, 6) and B(3, 2), then slope of AB is

- (A)  $\frac{3}{2}$                       (B)  $\frac{-2}{3}$                       (C)  $\frac{1}{3}$                       (D)  $\frac{-3}{2}$

2-  $\int_a^b 3t^2 dt =$

- (A)  $a^3 - b^3$                       (B)  $a^3 + b^3$                       (C)  $b^3 - a^3$                       (D)  $\frac{b^3 + a^3}{3}$

3- If  $\vec{OA} = \vec{a}$ ,  $\vec{OB} = \vec{b}$ , then  $\vec{AB} =$

- (A)  $\vec{a} - \vec{b}$                       (B)  $\vec{a} + \vec{b}$                       (C)  $\vec{b} - \vec{a}$                       (D)  $\vec{a} \cdot \vec{b}$

4- Minimum value of the function  $f(x) = x^2 + 2x - 3$  is at  $x =$

- (A) -3                      (B) 1                      (C) 0                      (D) -1

5- The range of  $f(x) = x^2$  is

- (A)  $(-\infty, 0)$                       (B)  $(-\infty, \infty)$                       (C)  $(-1, 0)$                       (D)  $(0, \infty)$

6-  $|\cos \alpha \underline{i} + \sin \alpha \underline{j} + 0 \underline{k}| =$

- (A) 0                      (B) -1                      (C) 2                      (D) 1

7- The length of tangent from (0, 1) to the circle  $x^2 + y^2 + 6x - 3y + 3 = 0$  is

- (A) 2                      (B) 3                      (C) 4                      (D) 1

8- (1, -3) is in the solution of region

- (A)  $x + y > 0$                       (B)  $x + y < 0$                       (C)  $x + y = 0$                       (D)  $x - y = 0$

9-  $\frac{d}{dx}(\sinh 2x) =$

- (A)  $2 \cosh 2x$                       (B)  $2 \sinh 2x$                       (C)  $-2 \cosh 2x$                       (D)  $-2 \sinh 2x$


10- Centre of the circle  $5x^2 + 5y^2 + 14x + 12y - 10 = 0$  is

- (A)  $\left(\frac{-7}{5}, \frac{-6}{5}\right)$                       (B)  $\left(\frac{7}{5}, \frac{6}{5}\right)$                       (C) (7, 6)                      (D) (7, -6)

(Turn over)

# Gujranwala Board-2019

(2)

- 11- If  $f(x) = \cos x$ , then  $f^2\left(\frac{\pi}{2}\right) =$
- (A) -1                      (B)  $-\frac{1}{2}$                       (C) 0                      (D) 1
- 12- Anti derivative of  $\cot x =$
- (A)  $\ln \cos x + c$                       (B)  $\ln \sin x + c$                       (C)  $-\ln \cos x + c$                       (D)  $-\ln \sin x + c$
- 13-  $\frac{d}{dx} (\cos^{-1} 3x) =$
- (A)  $\frac{3}{\sqrt{1-9x^2}}$                       (B)  $\frac{-3}{\sqrt{1-9x^2}}$                       (C)  $\frac{1}{\sqrt{1-9x^2}}$                       (D)  $\frac{-1}{\sqrt{1-9x^2}}$
- 14- Focus of parabola  $x^2 = -16y$  is
- (A) (0, -4)                      (B) (0, 4)                      (C) (4, 0)                      (D) (-4, 0)
- 15-  $\int_{-1}^0 \frac{1}{1+x^2} dx =$
- (A)  $\frac{\pi}{4}$                       (B)  $\frac{4}{\pi}$                       (C)  $-\frac{\pi}{4}$                       (D)  $-\frac{4}{\pi}$
- 16- Centroid of triangle with vertices A(2, 1), B(-1, 3) and C(-1, -4) is
- (A) (3, 1)                      (B) (0, 0)                      (C) (2, 2)                      (D) (-2, -5)
- 17-  $\int e^{\tan x} \sec^2 x dx =$
- (A)  $-e^{\tan x} + c$                       (B)  $e^{\tan x} + c$                       (C)  $e^{\tan^2 x} + c$                       (D)  $e^{\cos x} + c$
- 18- Distance between (1, 2) and (2, 1) is
- (A)  $\sqrt{3}$                       (B)  $\sqrt{5}$                       (C)  $\sqrt{2}$                       (D) 7
- 19- Equation of a straight line passing through P(-2, 3) and parallel to x-axis is
- (A)  $x = -2$                       (B)  $y = 3$                       (C)  $x = 3$                       (D)  $y = -2$
- 20-  $\frac{d}{dx} \left(\frac{1}{x^2}\right)$  at  $x = 1$  is 
- (A) -2                      (B) 2                      (C) 1                      (D) -1

313-(III)-419-20000

**SUBJECTIVE**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

**SECTION I**



2. Write short answers to any EIGHT questions:

- i- Determine whether  $f(x) = x\sqrt{x^2+5}$  is even or odd.
- ii- For the real valued function  $f(x) = \frac{2x+1}{x-1}$  find  $f^{-1}(x)$  and  $f^{-1}(-1)$
- iii- If  $f(x) = \begin{cases} x-1, & x < 3 \\ 2x+1, & 3 \leq x \end{cases}$  Find  $\lim_{x \rightarrow 3^-} f(x)$  and  $\lim_{x \rightarrow 3^+} f(x)$ .
- iv- Find the derivative of  $f(x) = c$  by first principle.
- v- Differentiate  $y = \frac{a+x}{a-x}$  w.r.t,  $x$
- vi- Find  $\frac{dy}{dx}$  if  $y = e^{x^2+1}$
- vii- Determine the values of  $x$ , for which  $f(x) = x^2 + 2x - 3$  is extreme.
- viii- Show that  $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$
- ix- If  $y = \sin^{-1} \frac{x}{a}$  then  $\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$
- x- Define a stationary point.
- xi- Define even function and give an example.
- xii- Find  $\frac{dy}{dx}$  if  $y = \tan h(x^2)$ .

3. Write short answers to any EIGHT questions:

- i- Use differentials, find  $\frac{dy}{dx}$  if  $x^2 + 2y^2 = 4$
- ii- Evaluate  $\int \cos 3x \cdot \sin 2x \, dx$
- iii- Evaluate  $\int \frac{\sin \theta}{1 + \cos^2 \theta} \, d\theta$
- iv- Integrate  $\tan^{-1} x$  w. r. t 'x'
- v- Evaluate  $\int e^x (\cos x + \sin x) \, dx$
- vi- Evaluate  $\int_{-1}^2 (x + |x|) \, dx$
- vii- Find area between x-axis and curve  $y = 4x - x^2$
- viii- Solve differential equation  $xy \, dy + y(x-1) \, dx = 0$
- ix- Define order of differential equation.
- x- Evaluate  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} \, dx$
- xi- Define corner point.
- xii- Graph the feasible region of  $3x - 2y \geq 6$

**(2 x 8 = 16)**

**(Turn over)**

**4. Write short answers to any NINE questions:**

(2 x 9 = 18)

- i- Show that points A(3, 1), B(-2, -3) and C(2, 2) are vertices of an isosceles triangle.
- ii- Define centroid of a triangle.
- iii- Find an equation of line through A(-6, 5) and having slope 7.
- iv- Convert into two intercept form  $2x - 4y + 11 = 0$
- v- Find centre and radius of circle  $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- vi- Determine whether the point P(-5,6) lies outside, on or inside the circle  $x^2 + y^2 + 4x - 6y - 12 = 0$
- vii- Write an equation of parabola with focus (-1, 0), vertex (-1, 2)
- viii- Find an equation of ellipse with centre (0, 0), focus (0, -3) and vertex (0, 4)
- ix- Define direction angles.
- x- If O is origin and  $\overline{OP} = \overline{AB}$ , find the point P where A and B are (-3, 7) and (1, 0) respectively.
- xi- Find a vector whose magnitude is 4 and is parallel to  $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$
- xii- Find a and b so that the vectors  $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$  and  $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k}$  are parallel.
- xiii- Find a scalar  $\alpha$  so that the vector  $2\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$  and  $3\mathbf{i} + \mathbf{j} + \alpha\mathbf{k}$  are perpendicular.

**SECTION II**

- 5- (a) Prove that  $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e$  5
- (b) Apply the Maclaurin series expansion to prove  $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$  5
- 6- (a) Evaluate the integral  $\int \frac{(a-b)x}{(x-a)(x-b)} dx$  5
- (b) Find an equation of the perpendicular bisector of the line segment joining the points A(3, 5) and B(9, 8) 5
- 7- (a) Find the integral  $\int_0^{\sqrt{7}} \frac{3x}{\sqrt{x^2+9}} dx$  5
- (b) Graph the feasible region of the inequalities and find the corner points: 5

$$\begin{aligned} x + y &\leq 5 \\ -2x + y &\geq 2 \\ x &\geq 0, \quad y \geq 0 \end{aligned}$$
- 8- (a) Show that the lines  $4x - 3y - 8 = 0$ ;  $3x - 4y - 6 = 0$ ;  $x - y - 2 = 0$  are concurrent and third line bisect the angle formed by first two. 5
- (b) Find equation of circle which passes through the points A(5, 10), B(6, 9) and C(-2, 3) 5
- 9- (a) Find the equation of 'Ellipse' with vertices (-1, 1); (5, 1) and foci (4, 1) and (0, 1) 5
- (b) Using vectors, find the area of triangle ABC whose vertices are A(1, -1, 1); B(2, 1, -1) and C(-1, 1, 2) 5

# Gujranwala Board-2019

Roll No. of Candidate \_\_\_\_\_

Mathematics

(INTER PART II)-419-(IV)

PAPER: II

GROUP:II

Time: 30 Minutes

Code: 8198

Marks: 20

## OBJECTIVE

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.




- 1- 1- The centre of the circle  $x^2 + y^2 - 6x + 4y + 13 = 0$  is  
(A) (3, 2) (B) (3, -2) (C) (2, 3) (D) (-2, -3)
- 2- If  $\alpha, \beta, \gamma$  be the direction angles of a vector then  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma =$   
(A) 2 (B) 0 (C) -1 (D) 1
- 3- The perpendicular distance of the line  $12x + 5y = 7$  from the origin is  
(A)  $\frac{7}{13}$  (B)  $\frac{13}{7}$  (C) 13 (D)  $\frac{1}{13}$
- 4-  $\int \tan^2 x \, dx$  is equal to  
(A)  $\tan x + x + c$  (B)  $\tan x - x + c$  (C)  $2 \tan x + c$  (D)  $2 \tan x + x + c$
- 5-  $\int \cot x \, dx =$   
(A)  $\operatorname{cosec}^2 x + c$  (B)  $-\operatorname{cosec}^2 x + c$  (C)  $\ln \sin x + c$  (D)  $\ln \cos x + c$
- 6- If  $y = \frac{1}{x^2}$  then  $\frac{dy}{dx}$  at  $x = -1$   
(A) 3 (B)  $\frac{1}{3}$  (C) 2 (D)  $\frac{1}{2}$
- 7- If  $f(x) = \frac{1}{x^2}$  ( $x \neq 0$ ), then  $f \circ f(x)$  is  
(A)  $x^4$  (B)  $x^2$  (C) 1 (D)  $\frac{1}{x^4}$
- 8- Angle between the vectors  $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and  $-\mathbf{i} + \mathbf{j} - 2\mathbf{k}$  is  
(A)  $30^\circ$  (B)  $45^\circ$  (C)  $90^\circ$  (D)  $60^\circ$
- 9- (1, 0) is the solution of the inequality  
(A)  $7x + 2y < 8$  (B)  $x - 3y < 0$  (C)  $10x + 5y < 6$  (D)  $-3x + 5y > 2$
- 10-  $\frac{d}{dx} (\ln 2x) =$   
(A)  $\frac{1}{2x}$  (B)  $\frac{1}{x}$  (C)  $-\frac{1}{2x}$  (D)  $2x$

(Turn over)



## Gujranwala Board-2019

(2)

- 11-  $\int_0^{\pi} \sec x \tan x \, dx =$  
- (A) 0 (B) 1 (C) -1 (D) -2
- 12- Eccentricity of an ellipse is
- (A)  $c = 1$  (B)  $e > 1$  (C)  $0 < e < 1$  (D)  $e = 0$
- 13- Order of the differential equation  $\frac{x^2 dy}{dx^2} + \frac{dy}{dx} + 2x = 0$  is
- (A) 0 (B) 1 (C) 2 (D) 3
- 14-  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n} =$
- (A)  $e$  (B)  $c^2$  (C)  $e^n$  (D) zero
- 15- The vertices of a triangle are  $(a, b - c)$ ,  $(b, c - a)$ ,  $(c, a - b)$  then its centroid is
- (A)  $\left(0, \frac{a+b+c}{3}\right)$  (B)  $\left(0, \frac{a-b-c}{3}\right)$  (C)  $(0, 0)$  (D)  $\left(\frac{a+b+c}{3}, 0\right)$
- 16- If  $f'(c) = 0$  then  $f(x)$  has relative maximum value at  $x = c$  if
- (A)  $f''(c) < 0$  (B)  $f''(c) > 0$  (C)  $f''(c) = 0$  (D)  $f'''(c) = 0$
- 17- The point of concurrency of altitudes of a triangle is called
- (A) centroid (B) orthocentre (C) in centre (D) circum centre
- 18- Slope of the line  $2x + y - 3 = 0$  is
- (A) 2 (B)  $\frac{2}{3}$  (C) -2 (D)  $-\frac{2}{3}$
- 19-  $y = \sin 3x$  then  $y_2$  is
- (A)  $9 \cos x$  (B)  $-9 \sin 3x$  (C)  $9 \sin 3x$  (D)  $-9 \cos 3x$
- 20- Axis of parabola  $x^2 = 4ay$  is
- (A)  $x = 0$  (B)  $y = 0$  (C)  $y = x$  (D)  $x = -y$

314-(IV)-419-19006

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I


(2 x 8 = 16)

2. Write short answers to any EIGHT questions:

- i- Define implicit function.
- ii- If  $f(x) = 2x + 1$  and  $g(x) = \frac{3}{x-1}$ ,  $x \neq 1$ , find  $f \circ g(x)$ .
- iii- Evaluate  $\lim_{x \rightarrow -1} \frac{x^3 - x}{x+1}$  by using algebraic technique.
- iv- Find  $\frac{dy}{dx}$  if  $y = (x-5)(3-x)$
- v- Find  $\frac{dy}{dx}$  if  $xy + y^2 = 2$
- vi- Differentiate  $\sin x$  w. r. t.  $\cot x$
- vii- Find  $\frac{dy}{dx}$  if  $y = \frac{x}{\ln x}$
- viii- Define the stationary point.
- ix- Find  $\frac{dy}{dx}$  if  $y = e^{-2x} \sin 2x$
- x- Differentiate  $\cot^{-1} \frac{x}{a}$  w. r. t.  $x$
- xi- Find  $y_2$  if  $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Using differentials find  $\frac{dx}{dy}$  if  $x^2 + 2y^2 = 16$
- ii- Define first order differential equation.
- iii- Evaluate  $\int \tan^2 x \, dx$
- iv- Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} \, dx$
- v- Evaluate  $\int \sin^{-1} x \, dx$
- vi- Evaluate  $\int \frac{e^x(1+x)}{(2+x)^2} \, dx$
- vii- Evaluate  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t \, dt$
- viii- Find the area between  $x$ -axis and curve  $y = \sin 2x$  from  $x = 0$  to  $x = \frac{\pi}{3}$
- ix- Solve the differential equation  $\frac{x^2+1}{y+1} = \frac{x}{y} \frac{dy}{dx}$  ( $x, y > 0$ )
- x- Evaluate  $\int x^2 \ln x \, dx$
- xi- Define problem constraints.
- xii- Graph the solution set of linear inequality  $3y - 4 \leq 0$  in  $xy$ -plane.

**4. Write short answers to any NINE questions:**

(2 × 9 = 18)

- i- Find the point that divides the join of A(-6, 3) and B(5, -2) in the ratio 2 : 3 internally.
- ii- A point P(5, 3) is in xy-coordinates system. Axcs are rotated through angle 45°. Find the new point P(X, Y)
- iii- Find an equation of line passing through (2, 3), having slope -1.
- iv- Find the point of intersection of the lines  $x + 4y - 12 = 0$  and  $x - 3y + 3 = 0$
- v- Find the centre and radius of the circle  $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
- vi- Determine the length of tangent drawn from point (-5, 4) to the circle  $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
- vii- Find the focus and directrix of the parabola  $x^2 = 4(y - 1)$
- viii- Find the centre and eccentricity of the ellipse  $\frac{(2x-1)^2}{16} + \frac{(y+2)^2}{16} = 1$
- ix- Define scalar product of two vectors.
- x- Find a vector of length 5 in the direction opposite to that of  $\underline{y} = \hat{i} - 2\hat{j} + 3\hat{k}$
- xi- Find a vector perpendicular to the plane containing vectors  $\underline{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ ,  $\underline{b} = 4\hat{i} + 3\hat{j} - \hat{k}$
- xii- A force  $\underline{F} = 2\hat{i} + \hat{j} - 3\hat{k}$  is acting at a point A(1, -2, 1). Find the moment of  $\underline{F}$  about point B(2, 0, -2)
- xiii- What are direction angles of a vector?

**SECTION H**

- 5- (a) If  $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ K, & x = 2 \end{cases}$  Find value of K so that f(x) is continuous at x = 2 5
- (b) Find  $\frac{dy}{dx}$  if  $y = \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}$ ,  $x \neq 0$  5
- 6- (a) Evaluate  $\int \frac{1+4x}{(x-3)(x^2+4)} dx$  5
- (b) If (4, -2), (-2, 4) and (5, 5) are vertices of a triangle, find the co-ordinates of its 'Incentre'. 5
- 7- (a) Evaluate  $\int_0^{\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{2\cos^2\theta} d\theta$  5
- (b) Graph the solution region and find the corner points of  $3x + 2y \geq 6$ ;  $x + y \leq 4$ ;  $x \geq 0$ ,  $y \geq 0$  5
- 8- (a) Show that the line  $2x + 3y - 13 = 0$  is tangent to the circle  $x^2 + y^2 + 6x - 4y = 0$  5
- (b) Prove that the angle in a semi-circle is a right angle. 5
- 9- (a) Show that an equation of parabola with focus at  $(a\cos\alpha, a\sin\alpha)$  and directrix at  $x\cos\alpha + y\sin\alpha + a = 0$  is  $(x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$  5
- (b) Find the volume of the tetrahedron whose vertices are A(2, 1, 8), B(3, 2, 9), C(2, 1, 4), D(3, 3, 10) 5

# Gujranwala Board-2018

**Mathematics**  
**Time: 30 Minutes**

**(INTER PART II)-418-(I)**

**PAPER: 11**

**Code: 8191**

**Marks: 20**

**OBJECTIVE**


**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.



- 1- 1- Range of  $f(x) = x^2 + 1$  is  
 (A)  $\mathbb{R}$  (B)  $\mathbb{R} - \{1\}$  (C)  $\mathbb{R} - \{-1\}$  (D)  $\{1, \infty\}$
- 2-  $\frac{e^x - e^{-x}}{2} =$   
 (A)  $\sin x$  (B)  $\cos x$  (C)  $\sinh x$  (D)  $\cosh x$
- 3-  $\frac{d}{dx} \sqrt{x} =$   
 (A)  $\sqrt{1}$  (B) 1 (C)  $\frac{1}{2} \sqrt{x}$  (D)  $\frac{1}{2\sqrt{x}}$
- 4-  $\frac{d}{dx} \cos x^2 =$   
 (A)  $\sin x^2$  (B)  $\sin x^2$  (C)  $-2x \sin x^2$  (D)  $-2x \sin x$
- 5- If  $f(x)$  has maximum value at  $x = c$ , then  $f'(c) = 0$  but  $f''(c)$  is  
 (A) negative (B) positive (C) zero (D) undefined
- 6-  $\frac{d}{dx} e^{f(x)} =$   
 (A)  $e^{f(x)}$  (B)  $f(x) e^{f(x)}$  (C)  $e^{f(x)} f'(x)$  (D)  $f(x) e^{f(x)}$
- 7-  $\int 2^x dx =$   
 (A)  $\frac{2^{x+1}}{x-1}$  (B)  $x 2^{x-1}$  (C)  $2^x \ln 2$  (D)  $\frac{2^x}{\ln 2}$
- 8-  $\int e^{ax} (af(x) + f'(x)) dx =$   
 (A)  $e^{ax} \cdot af(x)$  (B)  $e^{ax} \cdot f'(x)$  (C)  $e^{ax} \cdot f(x)$  (D)  $e^{ax} \cdot af'(x)$
- 9-  $\int (\ln x) \frac{1}{x} dx =$   
 (A)  $(\ln x)^2$  (B)  $\frac{(\ln x)^2}{2}$  (C)  $\frac{1}{x^2}$  (D)  $-\frac{1}{x^2}$
- 10-  $\int_0^1 \frac{1}{1+x^2} dx =$   
 (A) 1 (B)  $\frac{\pi}{4}$  (C) 0 (D)  $\frac{\pi}{2}$

## Gujranwala Board-2018

(2)

- 11- Coordinates of mid-point of A(-1, 4), B(6, 2)
- (A) (-7, 2)                      (B) (7, -2)                      (C)  $\left(\frac{5}{2}, 3\right)$                       (D)  $\left(3, \frac{5}{2}\right)$
- 12- If  $m_1, m_2$  are slopes of perpendicular lines, then  $m_1 m_2 =$  
- (A) 0                      (B) -1                      (C) 1                      (D) undefined
- 13- If a line meets x and y axes at 2, 3 units, then its equation is
- (A)  $2x + 3y = 0$                       (B)  $3x + 2y = 0$                       (C)  $\frac{x}{2} + \frac{y}{3} = 0$                       (D)  $\frac{x}{2} + \frac{y}{3} = 1$
- 14- If P(7, -2) lies on circle with centre (-5, 3), then its radius is
- (A) 13                      (B)  $\sqrt{13}$                       (C) 17                      (D)  $\sqrt{17}$
- 15- To find optimal solution we evaluate the objective function at
- (A) one point                      (B) origin                      (C) some points                      (D) corner points
- 16- Length of Latus Rectum of Parabola  $x^2 = 5y$  is
- (A) 5                      (B) 20                      (C)  $\frac{5}{4}$                       (D) 10
- 17- For hyperbola value of eccentricity e is
- (A) 1                      (B) less than 1                      (C) greater than 1                      (D) 0
- 18- If  $a = b$  then equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  represents
- (A) ellipse                      (B) parabola                      (C) hyperbola                      (D) circle
- 19- Direction cosines of z-axis are
- (A) [1, 0, 0]                      (B) [1, 1, 1]                      (C) [0, 1, 0]                      (D) [0, 0, 1]
- 20- If  $\vec{u} = \vec{v}$ , then  $\vec{u} \cdot (\vec{v} \times \vec{w}) =$
- (A) 0                      (B) 1                      (C) -1                      (D) cannot be calculated

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SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

(2 x 8 = 16)

## 2. Write short answers to any EIGHT questions:

- i- Define implicit function also write one example.
- ii- For  $f(x) = \frac{x^3 - x}{x^2 + 1}$ , determine whether given function is even or odd
- iii- Prove that:  $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e$
- iv- Find by definition the derivative of  $x(x - 3)$  with respect to 'x'.
- v- Find the derivative of  $(x^2 + 5)(x^3 + 7)$  w.r.t. 'x'
- vi- If  $y = x^4 + 2x^2 + 2$  prove that  $\frac{dy}{dx} = 4x\sqrt{y-1}$
- vii- Differentiate  $x^2 + \frac{1}{x^2}$  w.r.t.  $x - \frac{1}{x}$
- viii- Calculate  $\frac{d}{dx}(\cos\sqrt{x} + \sqrt{\sin x})$
- ix- If  $f(x) = \ln(e^x + e^{-x})$ . Find  $f'(x)$
- x- Find  $\frac{dy}{dx}$  if  $y = (\ln \tan hx)$
- xi- Find  $y_2$  if  $x = a \cos \theta$ ,  $y = a \sin \theta$ .
- xii- Divide 20 into two parts so that the sum of their squares will be maximum.

## 3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Using differentials find  $\frac{dy}{dx}$  and  $\frac{dx}{dy}$ .  $x^4 + y^2 = xy^2$
- ii- Evaluate:  $\int \frac{1}{1 + \cos x} dx$ .
- iii- Evaluate:  $\int \frac{e^x}{e^x + 3} dx$ .
- iv- Evaluate:  $\int \tan^{-1} x dx$ .
- v- Evaluate:  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t dt$
- vi- Evaluate:  $\int_0^3 \frac{dx}{x^2 + 9}$
- vii- Find the area between the x-axis and the curve  $y = \cos \frac{1}{2}x$  from  $x = -\pi$  to  $\pi$ .
- viii- Define differential equation.
- ix- Solve the differential equation  $\frac{dy}{dx} = \frac{y}{x^2}$
- x- Solve  $(e^x + e^{-x}) \frac{dy}{dx} = e^x - e^{-x}$
- xi- Define corner point.
- xii- Graph  $3x - 2y \geq 6$  in  $xy$  - plane.

## 4. Write short answers to any NINE questions:

(2 x 9 = 18)

- i- By means of slope, prove that the points  $(-1, -3)$ ;  $(1, 5)$ ;  $(2, 9)$  are collinear.
- ii- Find an equation of horizontal line through  $(7, -9)$
- iii- Find an equation of the line through  $(5, -8)$  and perpendicular to the join of  $A(-15, -8)$ ,  $B(10, 7)$
- iv- Find the distance of the pt.  $(6, -1)$  from the line  $6x - 4y + 9 = 0$
- v- Find the lines represented by  $6x^2 - 19xy + 15y^2 = 0$
- vi- Find the focus and directrix of parabola  $x^2 = 5y$
- vii- Convert  $x + 8 - y^2 + 2y = 0$  into the standard form and find its vertex.
- viii- Find an equation of the ellipse with vertices  $(0, \pm 5)$ , and eccentricity  $\frac{3}{5}$ .
- ix- Find the focus and covertices of an ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$
- x- Decide whether the triples  $45^\circ, 45^\circ, 60^\circ$  are the direction angles of a vector or not.
- xi- Find the projection of vector  $\underline{a}$  along vector  $\underline{b}$  and projection of  $\underline{b}$  along  $\underline{a}$ ,  
if  $\underline{a} = 3\hat{i} + \hat{j} - \hat{k}$ ,  $\underline{b} = -2\hat{i} - \hat{j} + \hat{k}$
- xii- If  $\underline{y}$  is a vector for which  $\underline{y} \cdot \hat{i} = 0$ ,  $\underline{y} \cdot \hat{j} = 0$ ,  $\underline{y} \cdot \hat{k} = 0$ . Find  $\underline{y}$ .
- xiii- Verify that the vectors  $\underline{a}$  and  $\underline{b} \times \underline{a}$  are perpendicular to each other, if  
 $\underline{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ ,  $\underline{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$



## SECTION II

- 5- (a) Find the values 'm' and 'n' so that given function 'f' is continuous at
- $x = 3$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

- (b) Differentiate:
- $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$
- w.r.t.
- $x$

- 6- (a) Evaluate:
- $\int x^3 e^{5x} dx$

- (b) Find the angles of the triangle whose vertices are
- $A(-5, 4)$
- ,
- $B(-2, -1)$
- and
- $C(7, -5)$

- 7- (a) Evaluate:
- $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2 \theta \cot^2 \theta d\theta$

- (b) Minimize
- $f(x, y) = 3x + y$
- subject to the constraints

$$3x + 5y \geq 15, \quad x + 6y \geq 9, \quad x \geq 0, \quad y \geq 0$$

- 8- (a) Find length of the chord cut off from the line
- $2x + 3y = 13$
- by the circle
- $x^2 + y^2 = 26$

- (b) Find the angle between the following vectors:
- $\underline{u} = 2\hat{i} - \hat{j} + \hat{k}$
- ,
- $\underline{v} = -\hat{i} + \hat{j}$

- 9- (a) Find the centre, foci, eccentricity, vertices and equations of directrices of hyperbola

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

- (b) Find the volume of the tetrahedron whose vertices are

$$A(2, 1, 8), \quad B(3, 2, 9), \quad C(2, 1, 4), \quad D(3, 3, 0)$$